Backscatter Ionogram Inversion

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Abstract— **A Backscatter Ionogram, BSI, is a plot showing the group path or time delay against operating frequency when using ground-based swept frequency radar. In the case of ground backscatter the received signals are reflected from distant locations on the Earth's surface. The ionosphere is the medium through which both the transmitted and received signals traverse. A Backscatter ionogram contains useful information regarding the state of the ionosphere at the time and over the range of the returned signal, which could be a few thousand kilometers from the transmitter/receiver location. Backscatter ionograms differ from the more conventional vertical incidence ionograms, where the received signals are reflected from a region of the ionosphere vertically above the sounder location. Methods of inverting backscatter ionograms to obtain ionospheric profiles offer an important means of remote sensing the distant ionosphere and regions in which land and sea scatter occur. Thus, backscatter ionograms can play an important role in the frequency management systems of over-the-horizon radar (e.g., [1]).**

Index Terms—**backscatter, ionogram, inversion, radar.**

I. Introduction

IN this paper a new BSI inversion method is shown. Several of the earlier BSI inversion methods, of varying complexity, are based on inverting the leading edge of the backscatter echo trace to a single quasi-parabolic (QP) ionospheric layer [2]-[6]. With the aid of sophisticated ray-tracing techniques and high-speed computers a number of the inversion methods, have been tested using synthesized backscatter ionograms. Even using synthesized data a number of the methods appeared unstable. For example, [6] found the method of Rao [3] to be unstable. The method of Rao [3] consisted of choosing 3 points from the leading edge of the backscattered echo. An iterative method is then applied to determine the corresponding QP ionospheric layer. Reference [6] showed that by increasing the data points to 15 or more improved the stability. However the solutions were not always unique. The method in [5], proposed the use of values from the leading edge of the backscatter echoes to determine 6 parameters representing a spherically asymmetric ionospheric layer. However, [7] believed that the constraints imposed on the vertical and horizontal structures appeared too strong to be able to solve the inverted ionospheric profile using the method in [5]. Reference [8] examined inversion using the leading and trailing edges of synthesized backscattered traces to form single QP layers. Reference [8] found that the leading and trailing edges could be transformed to a family of normalized curves, which depend on the layer parameters. This approach [8] was applied to real data with some success. However, a difficulty with this technique is that the trailing edge on most real backscatter ionograms is not visible, thus, making it an unsatisfactory inversion method in practice. Reference [9] outlines a more general iterative inversion method based upon an adaptation of the tele-siesmic techniques developed by [10]. Reference [7] developed a similar technique to that of [9] for reconstructing the structure of the horizontally inhomogeneous ionosphere. However, their methods required educated, or very close, approximations to the true ionospheric profile as a required input. In practice this required input is not always available, thus, their techniques are of only limited value. This study is only interested in inversion methods based upon multi-frequency sounding schemes. Inversion based upon schemes involving elevation-scan backscatter sounding, for example by [11] and [12], have not been considered.

For a BSI inversion method to be deemed appropriate for practical purposes it must be robust and the constraints within the method must be flexible enough so as to accommodate the data from actual backscatter sounding echoes.

This new BSI inversion technique is an extension to that of Rao [3] and [6]. This new inversion technique requires data from the leading edge of each of the layer echo traces on the backscatter ionograms and this data is then inverted to form a multiple quasi-parabolic segment ionospheric profile, using the quasi-parabolic segment model, abbreviated as the QPS model, and derived by [13]. The QPS model consists of 5 QPS, which represent the E, F1 and F2 ionospheric layers.

As mentioned by [14], traces, or echoes, observed on backscatter ionograms depend on the radar system characteristics and ionospheric characteristics as well as the backscattering properties of the ground. Important radar characteristics are the wavelength, pulse length, antenna beamwidth, antenna gain, sensitivity, transmitted and received power. If the radar characteristics are known then the backscatter ionogram becomes a function of the reflecting and scattering properties of the ionosphere and the ground. Throughout this investigation the radar characteristics are assumed.

This inversion technique derives quasi-parabolic ionospheric layer parameters from the echo traces on the BSI ionograms. At least 3 data points are required from each of the layer echo traces. The maximum or peak frequency of each of the ionospheric layer echo traces, is also a required input.

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II. THE QUASI-PARABOLIC SEGMENT QPS MODEL

The QP layer defined by [15] is given by

$$N_e = \begin{cases} N_m\left[1-\left(\dfrac{r-r_m}{y_m}\right)^2\left(\dfrac{r_b}{r}\right)^2\right], & r_b < r < r_m\left\{\dfrac{r_b}{r_b - y_m}\right\}, \\ 0, & otherwise \end{cases}$$

where N_e is the electron density at a radial distance r from the Earth's center,

N_m is the maximum electron density at the radial height r_m,

r_b is the radial base height of the ionospheric layer and

y_m is the layer semi-layer thickness.

The QPS model, shown in [13], is made up of QP segments where a QPS is used to describe each of the ionospheric layers and a QPS representing the joining segment, used to smoothly join the QP layers together. The equations describing the QPS ionospheric model may be written as

$$N_E = a_E - b_E\left(1-\frac{r_E}{r}\right)^2 \qquad \text{E layer}$$

$$N_j = a_E - b_j\left(1-\frac{r_E}{r}\right)^2 \qquad \text{Joining layer}$$

$$N_F = a_F - b_F\left(1-\frac{r_F}{r}\right)^2 \qquad \text{F layer.}$$

Where $a = N_m$ and $b = N_m(r_b / y_m)^2$ and

$$b_j = -r_F b_F\left(1-\frac{r_F}{r_c}\right)\Big/ r_E\left(1-\frac{r_E}{r_c}\right)$$

where the joining point, r_c, joining the joining layer to the F layer may be written as

$$r_c = \frac{r_F b_F\left(\dfrac{r_F}{r_E}-1\right)}{a_F - a_E + b_F\left(\dfrac{r_F}{r_E}-1\right)}$$

If a F1 layer were present then another QPS joining layer between the F1 and F2 QPS layers would be required.

Reference [15] shows exact expressions for calculating ray parameters such as the group path, P', for propagation in a spherically stratified ionosphere consisting of a single QP layer. Where the equation for calculating the group path may be written as

$$P' = 2\left[\int_{r_\circ}^{r_b}\frac{r\,dr}{r\sqrt{r^2 - r_\circ^2\cos^2\beta}} + \int_{r_b}^{r_t}\frac{r\,dr}{\sqrt{Ar^2 + Br + C}}\right] \qquad (1)$$

where r_t is the radial height, from the Earth's center, at which the ray is reflected and

$r^2\mu^2 - r_\circ^2\cos^2\beta = Ar^2 + Br + C$

where μ is the refractive index and β is the elevation angle.

Reference [15] shows explicit equations for the ray parameters in their QPS model for a spherically stratified ionosphere, where if the ray propagates through n segments the group path may be written as

$$P' = 2\left[\sqrt{r_b^2 - r_\circ^2\cos^2\beta} - r_\circ\sin\beta + \sum_n\left(U_n - L_n\right)\right]$$

where U_n and L_n represent the values of the integral in equation (1) at the upper and lower bounds.

III. THE NEW BSI INVERSION TECHNIQUE

This method of inversion of the BSI ionogram echo traces requires the determination of the three layer parameters for each of the quasi-parabolic layers, namely the critical frequency f_c, base height of the ionosphere r_b, and height of maximum electron density r_m.

The inversion technique begins with inversion of the BSI ionogram E layer echo trace. Once the parameters for the E layer have been calculated the inversion technique begins on the F1 layer echo trace using the E layer parameter results as well as the QPS equations for the joining layer, which smoothly joins the E layer and the F1 layer. The process is repeated for the F2 layer where the layer parameters already determined for the E and F1 layers as well as the equations for the joining layer that joins the F1 and F2 layers are required for the inversion process.

The inversion technique requires n (where n is at least 3) data points from each of the layer echo traces, where the n group paths are chosen $P_1', P_2' \ldots P_n'$ corresponding to the n frequencies f_1, $f_2 \ldots f_n$ respectively. The method then sets out to find a set of values for the layer parameters, which yield to within a specified accuracy, to the data points chosen. To accomplish this an initial guess of the ray parameters (f_c, r_b, r_m) is made.

The inversion method in [3] was found to be unstable in that it was very sensitive to the initial guess ionospheric parameters. This problem was solved with the inclusion of the maximum operating frequency of each of the layer echo traces as a required input. The maximum operating frequency of the layer echo trace, $f_{\max}$ together with the L value, described in [8], is used to determine the initial guess ionospheric parameters, which produced the leading edge of the ionospheric layer echo trace.

The elevation angles for ray paths where the leading and trailing edge meet (i.e., at the peak of the echo) is a minimum. For convenience let this elevation angle $\beta_\circ = 0^\circ$. Then, using the maximum operating frequency of the layer echo trace and an educated guess of the peak, or critical frequency f_c, of the layer, a value of the layer height r_m and the base height r_b of the ionospheric layer can be determined using the equations:

$$r_m = \sqrt{\frac{r_\circ^2}{\left(1-\dfrac{f_c^2}{f_{\max}^2}\right)}}$$

$$y_m = \frac{(r_m - r_\circ)}{L}$$

$$r_b = r_m - y_m$$

The L value as well as f_c can be incremented so that the process homes in to the best possible solution.
The elevation angles $\beta_1, \beta_2, \ldots, \beta_n$ at each of the n data points are then determined such that $\partial P'/\partial\beta = 0$. These computed values of elevation together with the new layer parameters are then substituted into the analytic expressions for the group path for the quasi-parabolic layer. Letting the computed minimum group paths, which most likeably will differ from the real values $P_1', P_2', \ldots, P_n'$, be $P_{c1}', P_{c2}', \ldots, P_{cn}'$, respectively, and the corresponding differences between the real and computed values be $\Delta P_1', \Delta P_2', \ldots, \Delta P_n'$, respectively. Then, to a first approximation the amount $\Delta f_c, \Delta r_b$ and Δr_m by which the assumed layer parameters should be incremented, so that $\Delta P_1', \Delta P_2', \ldots, \Delta P_n'$ are a minimum is given by (refer also to [6])

$$\Delta P'(f) = \sum_i \frac{\partial P'(f)}{\partial x_i} \delta\, x_i$$

Re-written in matrix form as

$$[\Delta P'] = \left[\frac{\partial P'}{\partial x_i}\right][dx_i]$$

or as $Z = ST$

where $Z = [\Delta P']$ is a (1,n) matrix,

$S = \left[\frac{\partial P'}{\partial x_i}\right]$ is a (3,n) matrix,

$T = [dx_i]$ is a (3,1) matrix.

The inversion of this is $T = [S_T S]^{-1} S_T Z$

Where S_T is the transpose matrix of S and the matrix $S_T S$ is a square matrix capable of inversion.
The assumed layer parameter values are then incremented by $\Delta f_c, \Delta r_b$ and Δr_m. The entire procedure begins again with the new assumed values until the differences in group path $\Delta P_1', \Delta P_2' \ldots \Delta P_n'$ converge to a small specified minimum, thereby obtaining the final solution of the layer parameters for that ionospheric layer.
Once the layer parameters have been calculated the next ionospheric layer parameters are then solved for using the technique above and the layer parameters already determined. For example let us assume that the E layer parameters r_{bE}, r_E, foE have already been evaluated using the method shown above. The peak operating frequency of the F1 layer echo trace, $f_{F1\max}$, may be determined directly from the BSI ionogram and at least 3 data points are required from the leading edge of the F1 layer echo trace. The unknowns to be solved for are $foF1$, r_{bF1} and r_{F1}.

Two additional QP segments are now involved where one represents the joining layer, which smoothly joins the E and F1 layers and the QPS layer representing the F1 layer from the peak of the F1 layer down to the point where these two layers are smoothly attached. The joining layer is made up of the E and F1 layer parameters (refer to [13]). Thus 3 QP segments are required to determine the F1 layer parameters using the inversion technique shown here. The equation for the group path in determining the F1 layer parameters contains the parts of the ray path in

1. free space (region between transmitter and base of the ionosphere),
2. the QPS representing the E layer,
3. the joining QPS layer and,
4. the QPS representing the F1 layer.

The equation for the group path for the propagated ray paths being reflected from the F1 layer may be written as

$$P' = 2\left[\sqrt{r_b^2 - r_\circ^2 \cos^2\beta} - r_\circ \sin\beta + U_E - L_E + U_j - L_j + U_{F1} - L_{F1}\right]$$

Once the F1 layer parameters are known the F2 layer parameters maybe calculated using the same procedure. In all 5 QP segments are required to determine the F2 layer parameters.

IV. Inversion of a Synthesized BSI

To test the inversion technique and the accuracy of this method, a synthesized BSI with known QP layer parameters was used. The synthesized BSI, in Fig. 1, was determined using an analytic ray tracing program and having the ionospheric layer parameters below:-

$foE = 3.0$ MHz, $y_m E = 15$ km, $r_E = 100 + r_\circ$ km E layer
$foF1 = 5.0$ MHz, $y_m F1 = 30$ km, $r_{F1} = 150 + r_\circ$ km F1 layer
$foF2 = 8.0$MHz, $y_m F2 = 80$ km, $r_{F2} = 250 + r_\circ$ km F2 layer
Where $r_\circ = 6370$ Km (the radius of the Earth).

The inversion technique was then applied. The data points were chosen from each of the layer echo traces in Fig. 1. Where at least 3 points, from the leading edge of each layer echo traces, are required. The peak operating frequency from each of the layers is also a required input.
In this case the accuracy of the technique is tested. Thus, greater accuracy in the input data is required. In general the more data points the higher the accuracy.

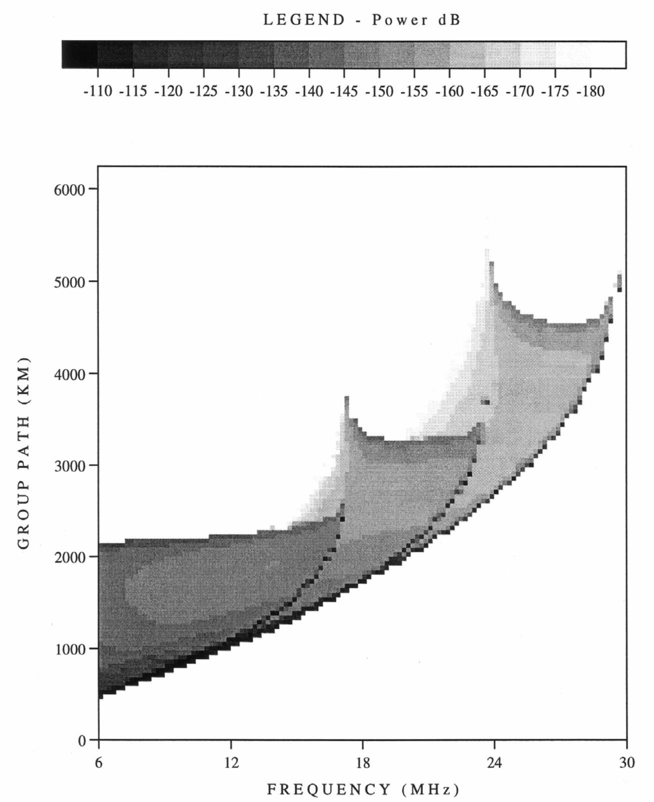

Fig 1. A typical daytime synthesized backscatter ionogram, with E, F1 and F2 layer echo traces.

The 8 data points (frequency [MHz], group path [km]) chosen from the leading edge of the E layer echo trace are shown in Table I and the maximum operating frequency of the E layer echo trace, $f_{E\max} = 17.0$ MHz.

TABLE I
THE 8 DATA POINTS CHOSEN FOR THE E LAYER

Data point	f_E [MHz]	P'_E [Km]	$\Delta P'_E$ [Km]
1	7.0	564.0323	-5.5×10^{-5}
2	8.0	654.6292	2.9×10^{-6}
3	9.0	750.1276	2.4×10^{-4}
4	10.0	851.8890	-2.3×10^{-4}
5	11.0	961.7783	-1.1×10^{-4}
6	12.0	1082.481	3.7×10^{-4}
7	13.0	1218.136	-2.5×10^{-4}
8	14.0	1375.754	5.4×10^{-5}

The $\Delta P'$ values show the difference in group path between the measured input data and the calculated results using our BSI inversion software. Once the BSI inversion software has undergone a set number of iterations or has homed into a result very close to the measured result, as in this example, the program automatically stops.

Table II shows the 5 data points chosen from the leading edge of the F1 layer echo trace and the maximum operating frequency of the F1 layer echo trace, $f_{F1\max} = 23.5$ MHz.

TABLE II
THE 5 DATA POINTS CHOSEN FOR THE F1 LAYER

Data point	f_{F1} [MHz]	P'_{F1} [Km]	$\Delta P'_{F1}$ [Km]
1	12.0	1009.608	1.2×10^{-4}
2	13.0	1110.457	1.5×10^{-4}
3	14.0	1216.541	-7.8×10^{-4}
4	15.0	1328.947	7.1×10^{-4}
5	16.0	1449.081	-1.9×10^{-4}

The 6 data points chosen from the leading edge of the F2 layer echo trace are shown in Table III and the maximum operating frequency of the F2 layer echo trace, $f_{F2\max} = 29.5$ MHz.

TABLE III
THE 6 DATA POINTS CHOSEN FOR THE F1 LAYER

Data point	f_{F2} [MHz]	P'_{F2} [Km]	$\Delta P'_{F2}$ [Km]
1	19.0	1838.218	1.9×10^{-4}
2	20.0	1973.166	-4.1×10^{-4}
3	21.0	2117.230	3.5×10^{-4}
4	22.0	2272.290	-2.8×10^{-4}
5	23.0	2440.870	2.1×10^{-4}
6	24.0	2626.486	-6.3×10^{-4}

The inversion technique produced the following layer parameters:-

$foE = 3.00$ MHz, $y_m E = 15.00$ km, $r_E = 100.00 + r_\circ$ km

$foF1 = 5.00$ MHz, $y_m F1 = 30.03$ km, $r_{F1} = 150.01 + r_\circ$ km

$foF2 = 8.00$ MHz, $y_m F2 = 79.97$ km, $r_{F2} = 249.98 + r_\circ$ km

Clearly, these values compare very well with the actual values of the layer parameters.
The final differences in group path $\Delta P'$ between the computed Group paths, determined using the backscatter inversion method and the given measured input group paths, at that particular frequency for the each of the layer echo traces are given in Tables I-III. The errors or differences in group path are very small, less than 1 meter.

V. CHECKING SENSITIVITY OF THE BSI INVERSION METHOD

The sensitivity of the backscatter ionogram inversion technique was examined in the following manner. The time delay (i.e., group path) in the resultant radar signal is normally binned into 50 km blocks the accuracy in the leading edge varies between 0 and 50 km. Thus, the next step is to round the synthesized data as if it were collected from a real BSI. The profiles and the associated backscatter ionograms

obtained using this rounded data are then compared to those obtained from the more accurate data. The data is rounded so that if it falls within a given range bin it automatically takes the mid point value of that range bin.
The values below in Table IV show the rounded data and the resultant differences between the group paths of the actual input values and those determined using the backscatter inversion method with the rounded data.

TABLE IV

Data point	f_E [MHz]	P'_E [Km]	$\Delta P'_E$ [Km]
1	7.0	575.0	8.8
2	8.0	675.0	17.4
3	9.0	775.0	20.8
4	10.0	875.0	17.7
5	11.0	975.0	5.9
6	12.0	1075.0	-17.4
7	13.0	1225.0	-6.6
8	14.0	1375.0	-19.6
Data points	f_{F1} [MHz]	P'_{F1} [Km]	$\Delta P'_{F1}$ [Km]
1	12.0	1025.0	14.6
2	13.0	1125.0	13.8
3	14.0	1225.0	7.8
4	15.0	1325.0	-4.4
5	16.0	1425.0	-24.1
Data points	f_{F2} [MHz]	P'_{F2} [Km]	$\Delta P'_{F2}$ [Km]
1	19.0	1825.0	-12.8
2	20.0	1925.0	2.3
3	21.0	2125.0	8.4
4	22.0	2225.0	3.7
5	23.0	2425.0	-14.5
6	24.0	2625.0	0.5

The following layer parameters were calculated using the backscatter inversion method and the rounded data.

$foE = 2.91$ MHz , $y_m E = 19.08$ km , $r_E = 95.43 + r_\circ$ km
$foF1 = 4.96$ MHz , $y_m F1 = 24.47$ km , $r_{F1} = 146.81 + r_\circ$ km
$foF2 = 8.06$MHz , $y_m F2 = 83.99$ km , $r_{F2} = 251.96 + r_\circ$ km

The red curve in Figure 2 shows the ionospheric profile having the above parameters.
Clearly, the difference in measured and calculated group paths $\Delta P'$ is larger in this case but still well within the 50 km bin size.

VI. Noisy Input Data

Checking the robustness of this method consisted of randomly adding/subtracting 50 km (corresponding to 1 range bin), or 0 km, in group path from each of the leading edge data points. Thus, testing the inversion method against noisier input data. The data points in Table 3 were chosen to test the inversion technique. The peak operating frequency for the E, F1 and F2 layers were 17.0, 23.5 and 29.5 MHz respectively.

TABLE V
Shows the Chosen Noisy Data Points

Data points	f_E [MHz]	P'_E [Km]	$\Delta P'_E$ [Km]
1	7.0	575.0	2.7
2	8.0	625.0	-39.6
3	9.0	825.0	62.8
4	10.0	875.0	8.5
5	11.0	925.0	-54.4
6	12.0	1125.0	21.0
7	13.0	1225.0	19.7
8	14.0	1425.0	15.6
Data points	f_{F1} [MHz]	P'_{F1} [Km]	$\Delta P'_{F1}$ [Km]
1	12.0	1075.0	60.6
2	13.0	1125.0	9.4
3	14.0	1225.0	3.1
4	15.0	1275.0	-59.5
5	16.0	1425.0	-29.8
Data points	f_{F2} [MHz]	P'_{F2} [Km]	$\Delta P'_{F2}$ [Km]
1	19.0	1875.0	55.2
2	20.0	1925.0	-27.3
3	21.0	2125.0	31.5
4	22.0	2225.0	-20.1
5	23.0	2375.0	-34.7
6	24.0	2575.0	-15.4

Using this data the backscatter ionogram inversion method produced the following QPS layer parameter results:

$foE = 2.94$ MHz , $y_m E = 19.49$ km , $r_E = 97.45 + r_\circ$ km
$foF1 = 4.99$ MHz , $y_m F1 = 24.77$ km, $r_{F1} = 148.65 + r_\circ$ km
$foF2 = 7.89$MHz , $y_m F2 = 60.21$ km , $r_{F2} = 240.84 + r_\circ$ km

The blue curve in Figure 2 shows the ionospheric profile having the above parameters.

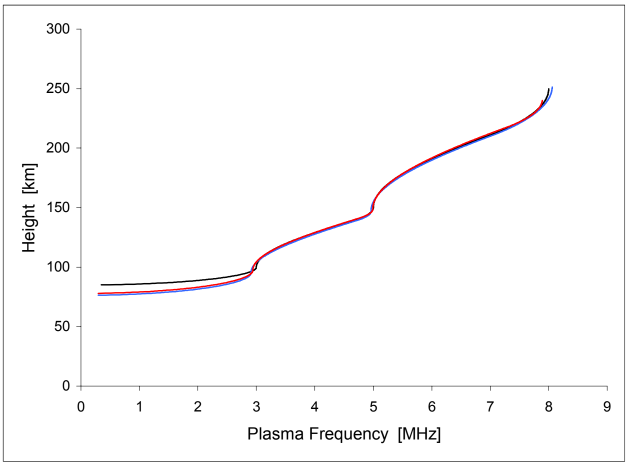

Fig 2. Three layer QPS ionospheric profiles where
Black curve – no noise, true ionospheric profile
Red curve – rounding and using mid value of range bin
Blue curve – rounding & plus/minus 50 km, or 0 km, random error

Figure 3 shows the BSI produced when using these layer parameters. Figure 4 shows the difference ionogram, highlighting the differences in signal strength between the ionograms in Fig 1 and 3. The differences are mainly in the leading edge of the echo traces.

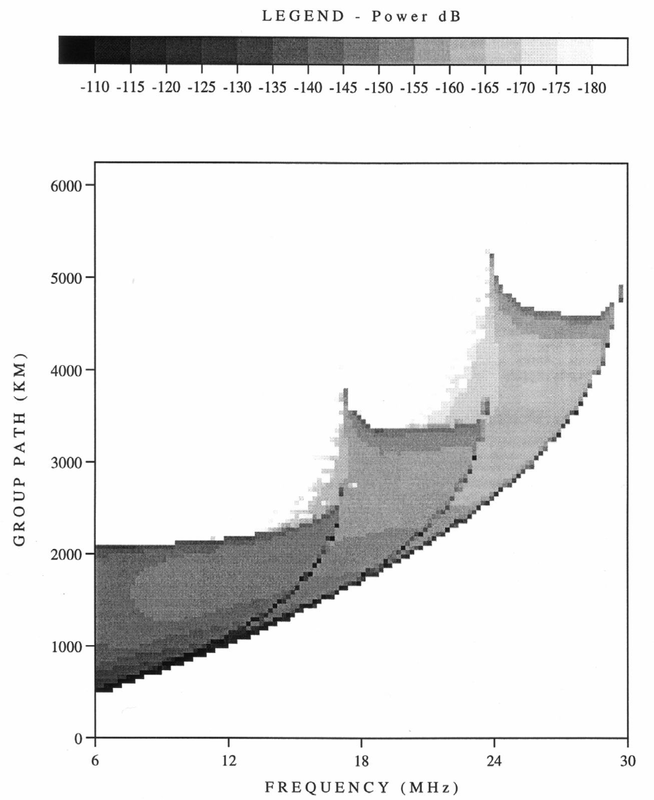

Fig 3. Synthesized backscatter ionogram, with the layer parameters determined using the rounded input data.

Figure 5 shows the difference in ground range between the ionogram in Fig 1 and Fig 3. This result is important for coordinate registration CR as it highlights the (frequency, group path) regions where CR may fall below acceptable operating levels. The results are very encouraging, as the difference in CR is less than 10 km, except at the leading and trailing edges where the CR difference is significantly worse.

Table 3 also shows the difference between the group paths of the actual input values and those determined using the backscatter inversion method with the rounded data with the $\pm$ 50 km random error. Clearly, the differences in group path are larger in this case but still well within the 100 km or 2 range bins.

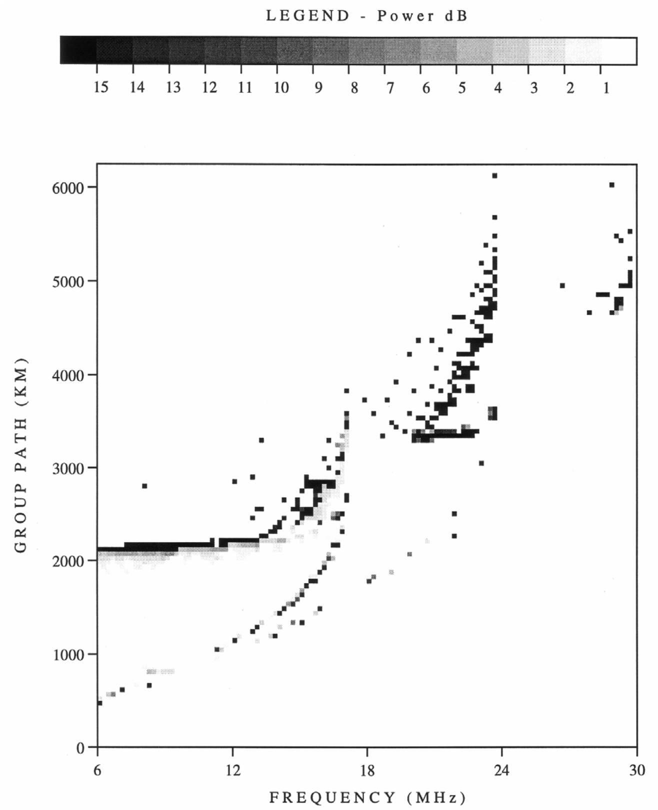

Fig 4. Difference ionogram showing the difference between the synthesized ionograms in Fig. 1 and Fig.2.

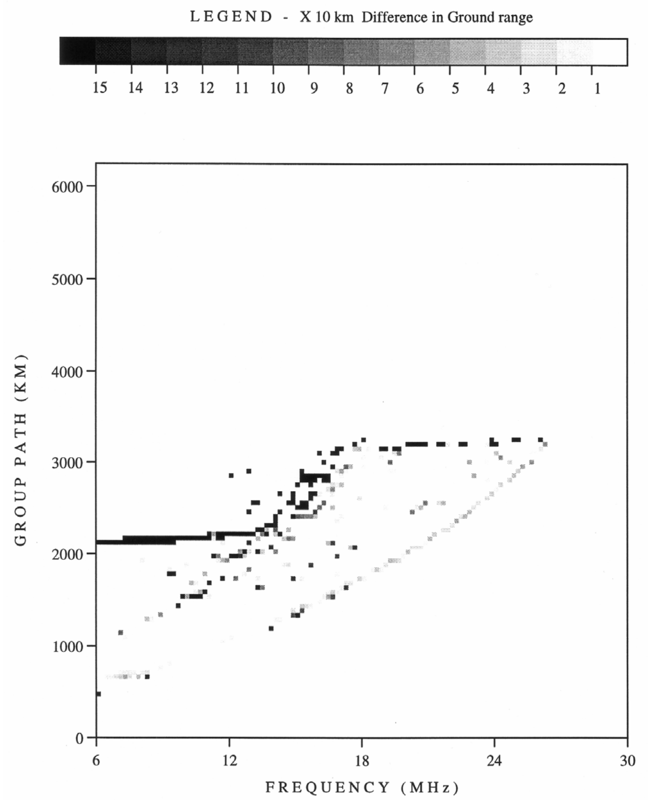

Fig 5. Shows the resultant difference in ground range when using the accurate input data to the Noisy input data, up to a range of 3500 km.

VII. CONCLUSION

The backscatter inversion technique developed here is robust and homes into the best possible solution. The results presented are encouraging even when using noisy data. The ground range difference plot in Fig 5 was encouraging especially for CR purposes, as the differences in ground range, with respect to group path, between the noisy and true input data was very small.

The program produces the results almost instantaneously. However, if computational time is of major importance then choosing only three points from each of the layer echo traces may be the way to go.

The backscatter ionogram inversion technique will attempt to fit the best possible QPS model parameters to the given input data.

How well this inversion technique performs, when using real backscatter data, requires further investigation.

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