Effects of non-unity refractive index on SuperDARN velocity estimates

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Abstract

Recent studies revealed that ionospheric ExB drift velocity estimates derived from SuperDARN Doppler shift data are on average by 25% smaller than those measured simultaneously by DMSP satellites positioned on the same field line. The 500-km altitude shift from the effective scattering volume to the satellite position leads to an 11-% increase in the E/B ratio, but this only accounts for less than a half of the observed effect. While the SuperDARN data processing algorithms assume that HF scatter occurs in a free space, our theoretical calculations and numerical ray tracing show that accounting for the non-unity refractive index in the ionosphere can qualitatively and quantitatively explain the remainder of the discrepancy between two instruments.

Outline

- Problem formulation
- Theoretical analysis
- Model calculations
- Conclusions
- Future directions

Problem formulation

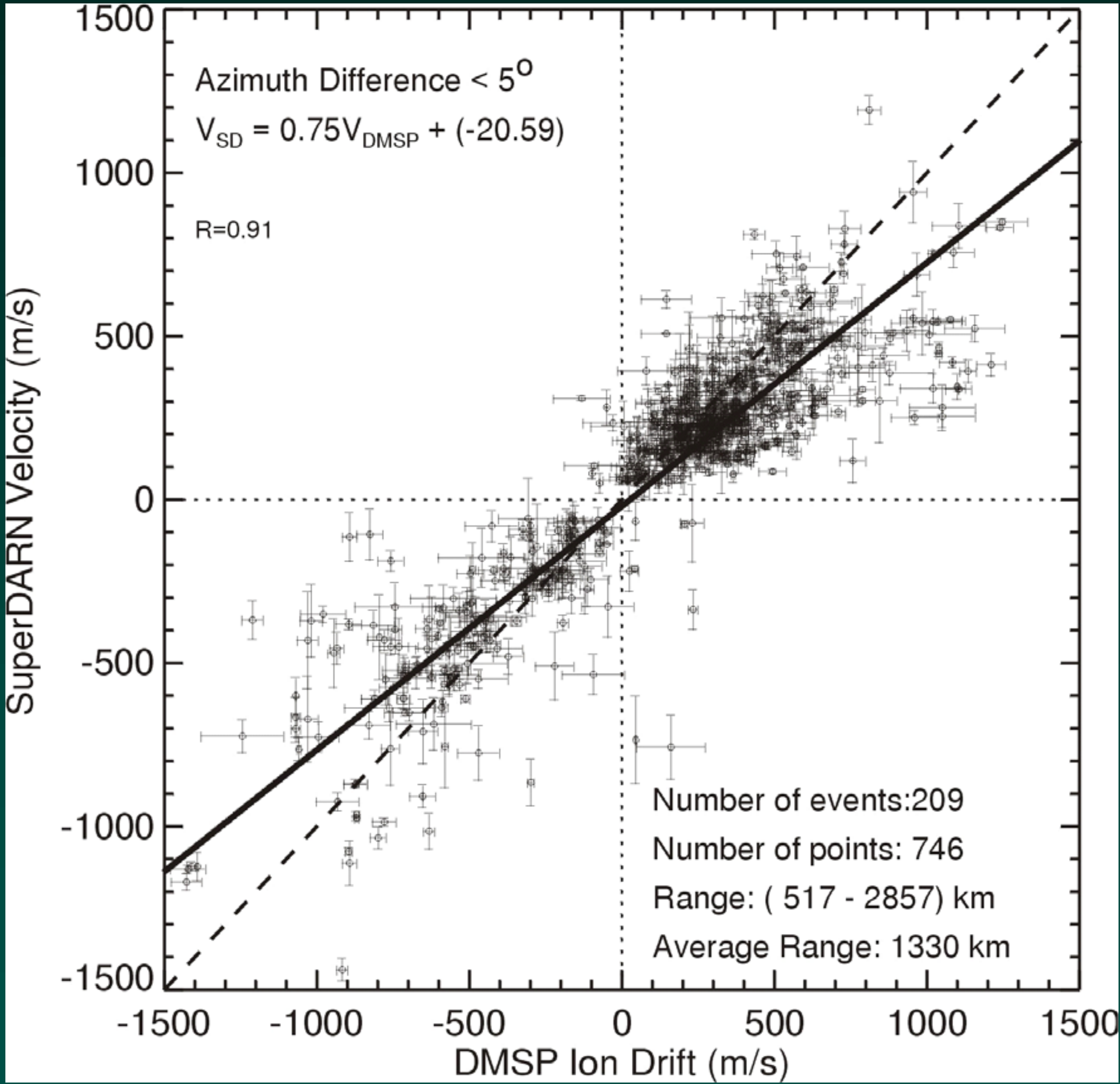
PHYSICS AND ENGINEERING PHYSICS

What is there to learn from comparing DMSP with SuperDARN?

J-P St-Maurice, R. Drayton, R. Choudhary, A. Kustov and S. Bansal

Institute of Space and Atmospheric Studies

The problem in a nutshell



1) SuperDARN is slower by 25 % on average. Why?

2) Lots of scatter. Why?

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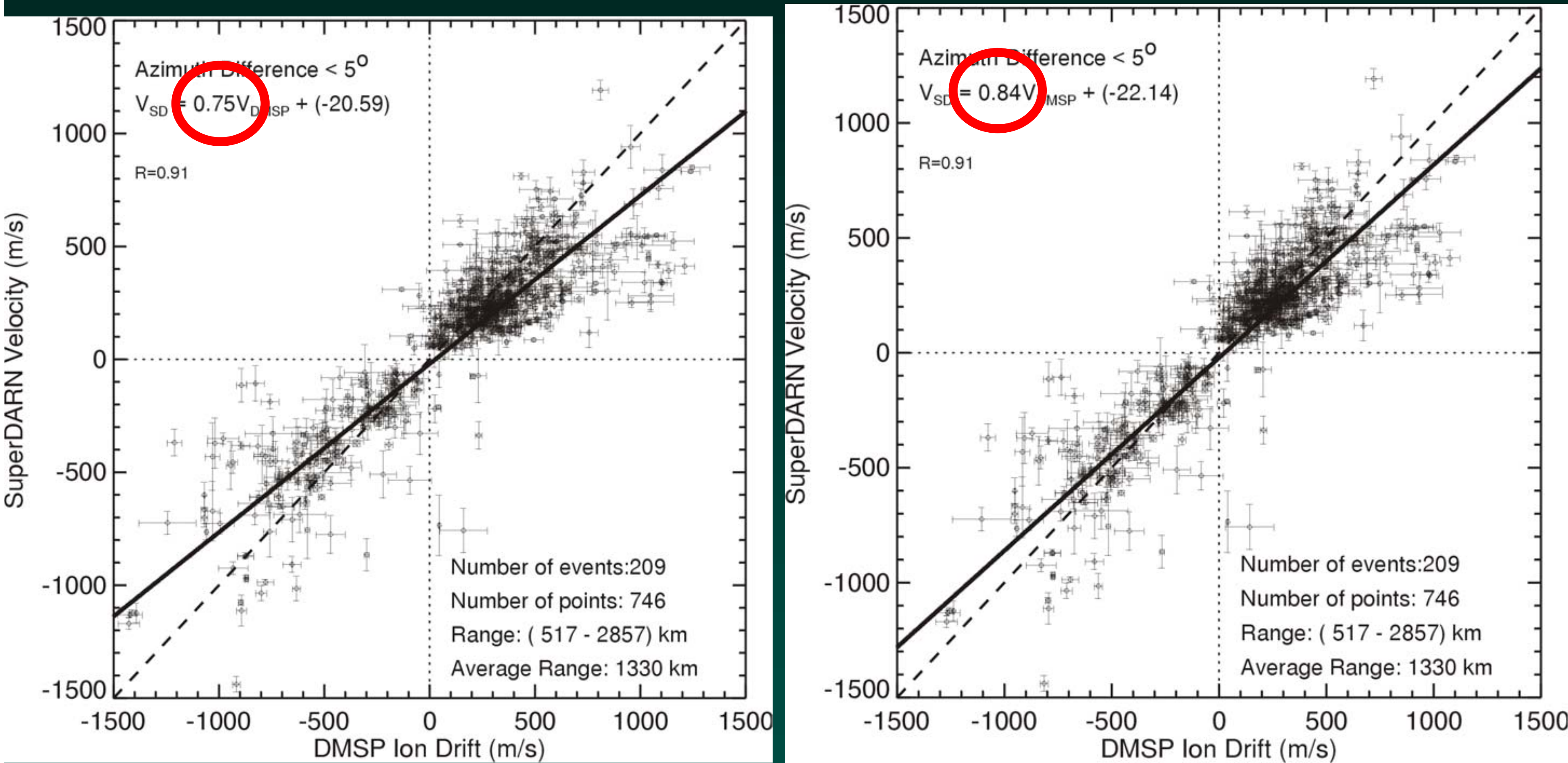
Second task: correct for E/B



Significant, as expected: about a 10% improvement in the slope. However, still bothersome disagreement.

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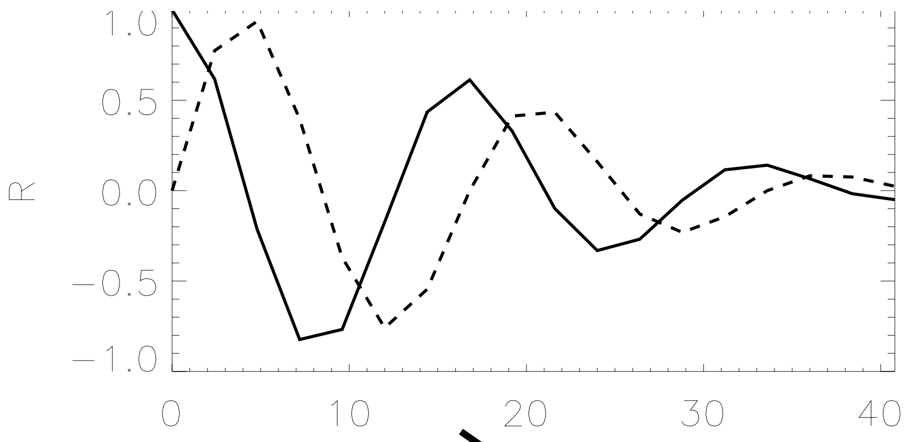
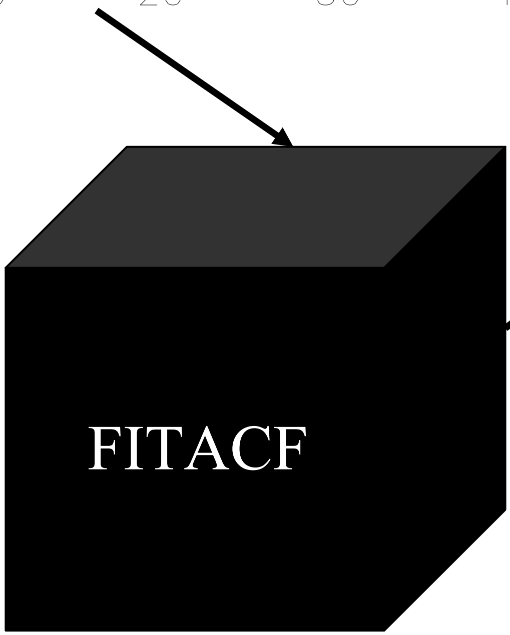
SuperDARN velocity



$$F_D = -2V_D / \lambda_0$$



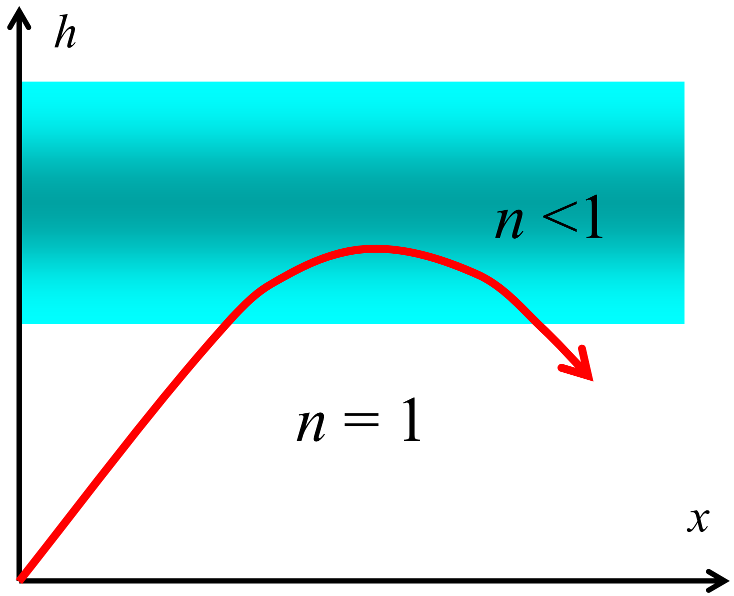
Is this correct?

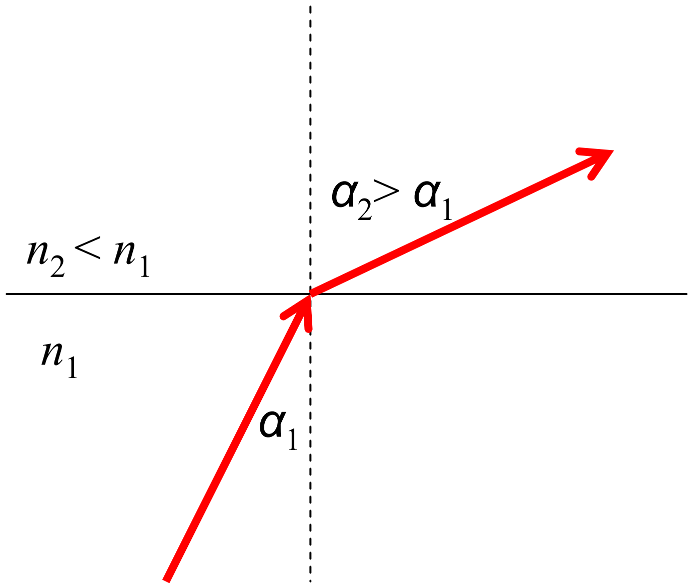
Theoretical analysis

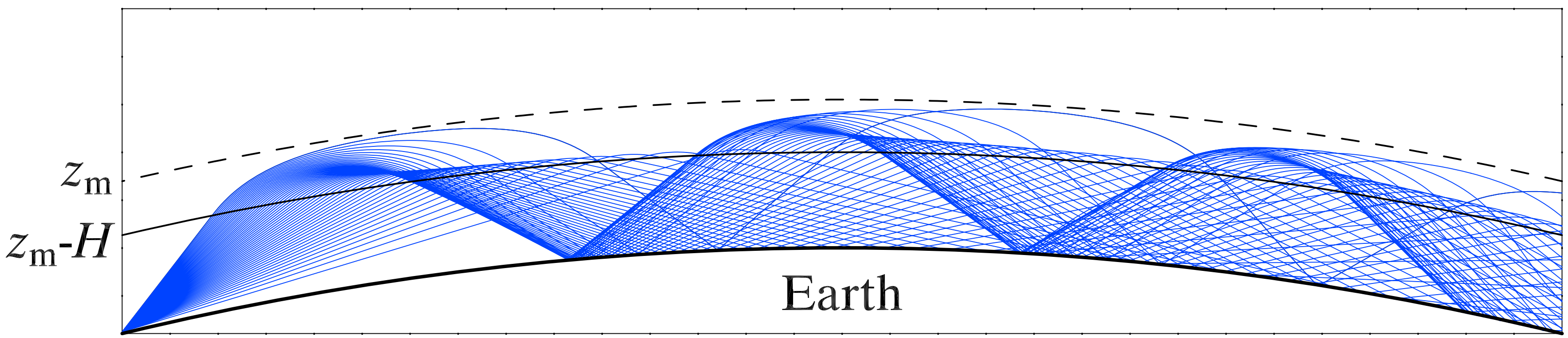
Ionospheric refraction

$$n^2 = 1 - \frac{f_N^2}{f_0^2}, \quad f_N \propto \sqrt{N_e}$$

$$n_1 \sin\alpha_1 = n_2 \sin\alpha_2$$







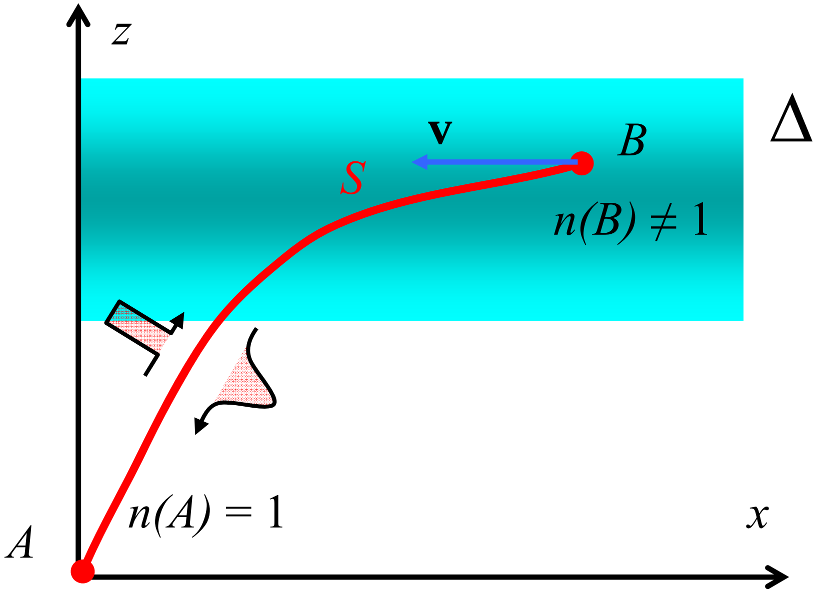
Group and phase velocities

$$n = \sqrt{1 - \frac{f_N^2}{f_0^2}} \leq 1$$

$$V_g = \frac{\partial \omega}{\partial k} = cn \leq c$$ Group path is overestimated!

$$V_p = \frac{\omega}{k} = \frac{c}{n} \geq c$$ What does it mean?

Doppler shift



$$\Delta\omega^{AB} = -k_0\left[\int_A^{B(t)} \frac{\partial n}{\partial t}\, ds + \frac{\partial B(t)}{\partial t} n(B)\right]$$

$$= -k_0\; n(B) V_{\parallel}^{AB}(B)$$

$$\Delta\omega^{AB} = \Delta\omega^{BA}$$

$$\Delta\omega = -k_0 \frac{\partial L_p}{\partial t}$$

$$\Delta\omega = -2k_0\, \boxed{n(B)}\, V_{\parallel}^{AB}(B)$$

$$L_p = \int_A^B n\; ds$$

Ginzburg, 1958

$$F_D = 2V_{app} / \lambda_0 = 2V_D n / \lambda_0$$

$$V_{app} = V_D n$$

$$n < 1$$

Velocity magnitude is underestimated!

By how much?

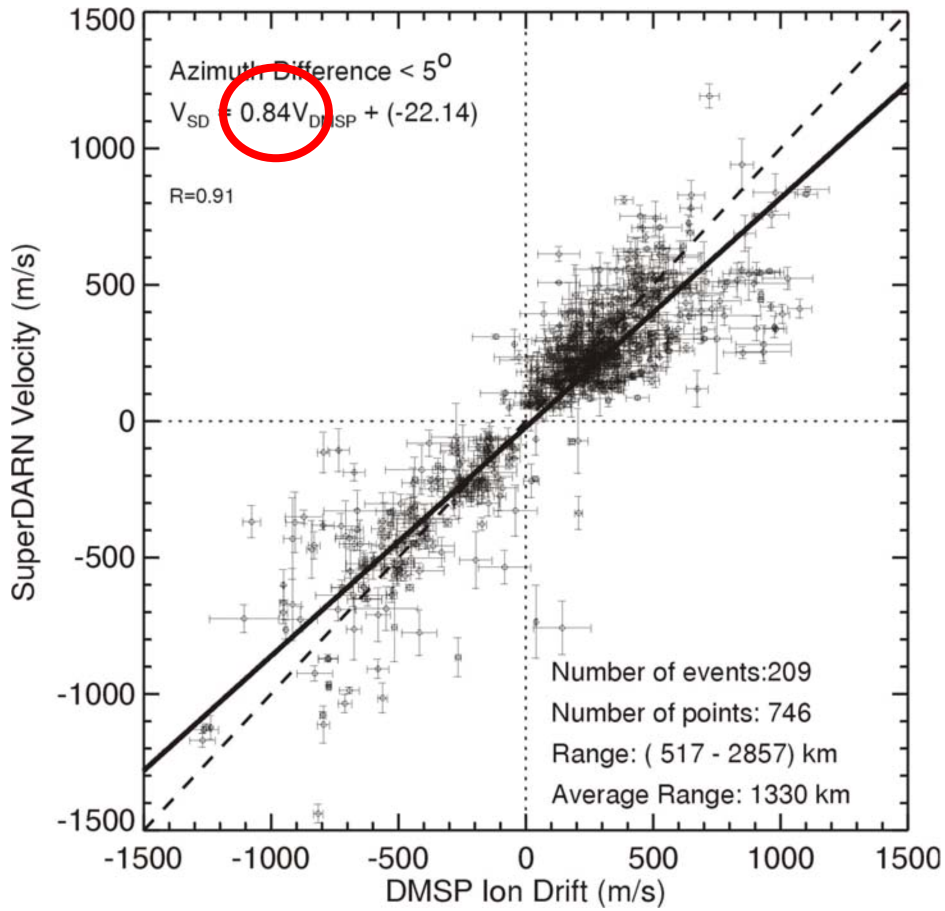
Rough estimates

$$n = \sqrt{1 - \frac{f_N^2}{f_0^2}}$$

f_0 =12-14 MHz

f_N^{max} =6-8 MHz

n^{min} =0.75-0.90



$$V_{app}=0.75\text{-}0.90\ V_{real}$$

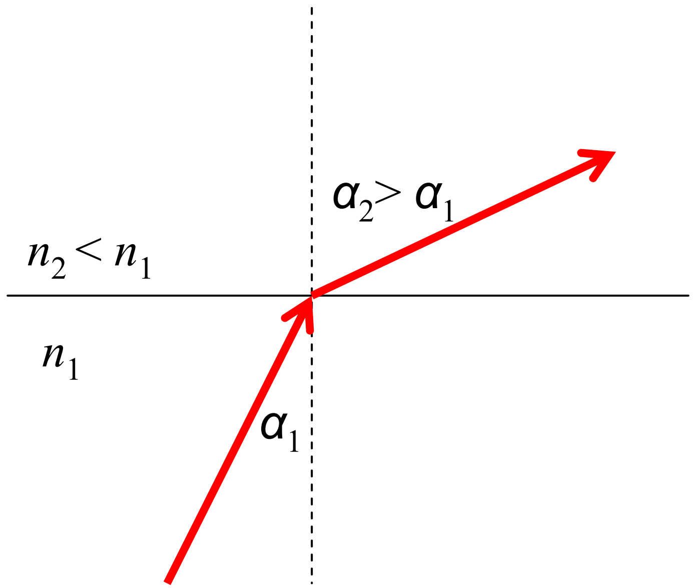
- This is the **maximum possible** velocity distortion for the given ratio f_N^{max} / f_0.
- However, in reality the scattered signal is formed by the whole **scattering volume** covering different altitudes with different n.
- Also, the scattering process is heavily affected by the **aspect conditions**.

Model calculations

Ray-tracing

- Simple numerical model
  - Parabolic layer
  - Flat ionosphere
  - No horizontal gradients
  - Constant magnetic field inclination

$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$



Anisotropic irregularities

Рытов, Кравцов и Татарский, 1978

рассмотрены в гл. V—VII.

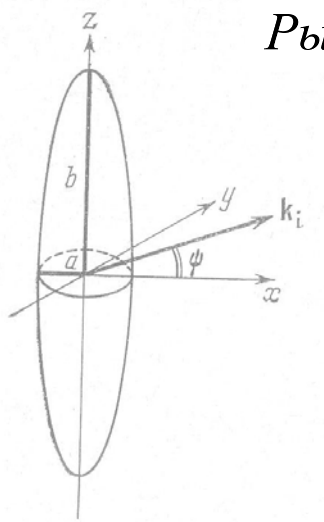


Рис. 29.

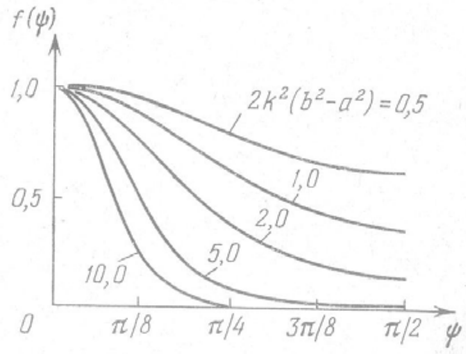


Рис. 30.

2. Обратное рассеяние на анизотропных (анизомерных) флуктуациях с гауссовой корреляционной функцией:

Scattered power diagram:

$$f(\psi) = \exp\left\{-2k^2\left(b^2 - a^2\right)\sin^2\psi\right\},\ a = \lambda/2$$

Dependence on fluctuation magnitude

In a recent paper, Walker et al. (1987) start with Booker's expression for backscatter cross-section

$$\sigma_B = r_e^2 \langle \Delta N^2 \rangle P_3(2kl, 2km, 2kn), \qquad (2.70)$$

where:

$\langle \Delta N^2 \rangle$ = the mean square fluctuation level of the electron density
r_e = the classical electron radius = 2.813×10^{-15} m
$P_3(k_\xi, l_\eta, m_\zeta)$ = the three-dimensional power spectrum of the irregularities
$k = 2\pi/\lambda$ = wave number of the transmitted signal,

and derives a general expression for the backscattered power (P_r) at the receiver as

$$P_r = r_e^2 \langle \Delta N^2 \rangle f_2(\theta_I) \frac{(l_p A_r G_t P_t g_t g_r \Delta\Phi_B)}{4\pi r'} \int_{\Delta\Psi} P_3(2k, 0, 2k\Psi)\, d\Psi, \qquad (2.71)$$

where:

$f_2(\theta_I) = \sin\theta / [\mu_p^2 f_1(\theta_I) \sin\{\theta_I + (\alpha - \alpha_o)\}]$
$f_1(\theta_I) = \frac{PQ}{P'Q'}$, the correction factor for focusing
l_p = spatial length of radar pulse
A_r = effective area of receiving antenna
G_t = gain of transmitting antenna
P_t = transmitter power output
μ_p = refractive index at the reflection level
g_r = normalized receiving antenna pattern
g_t = normalized transmitting antenna pattern

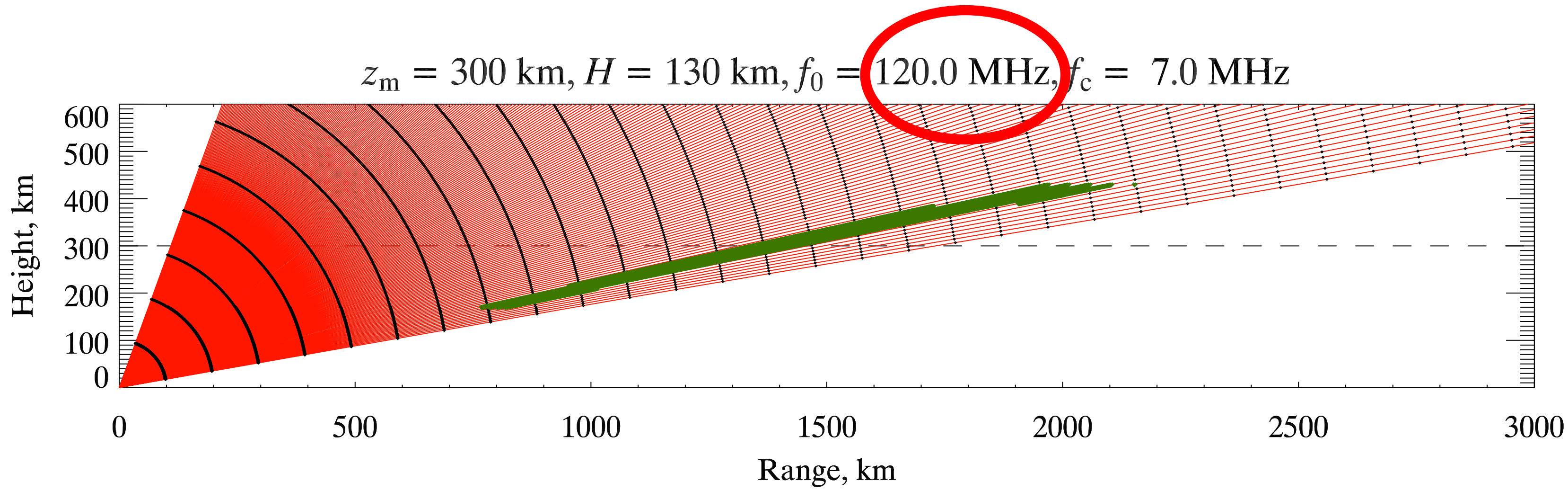
Hunsucker, 1991

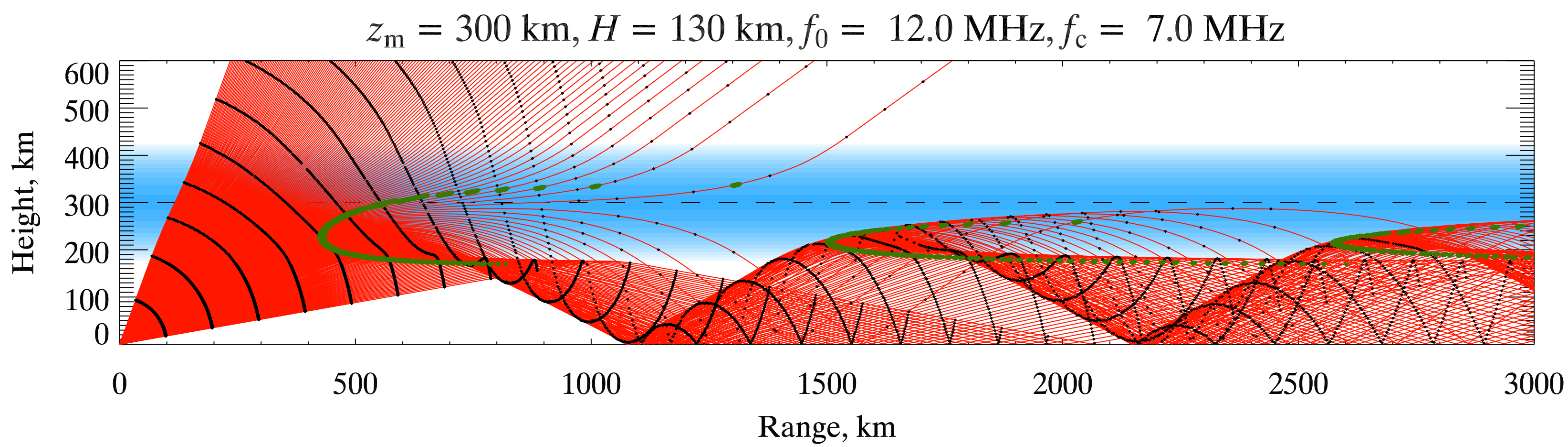
$$\frac{\langle \Delta N^2 \rangle}{N^2} = const$$

$$P_r \propto \langle \Delta N^2 \rangle \propto N^2$$

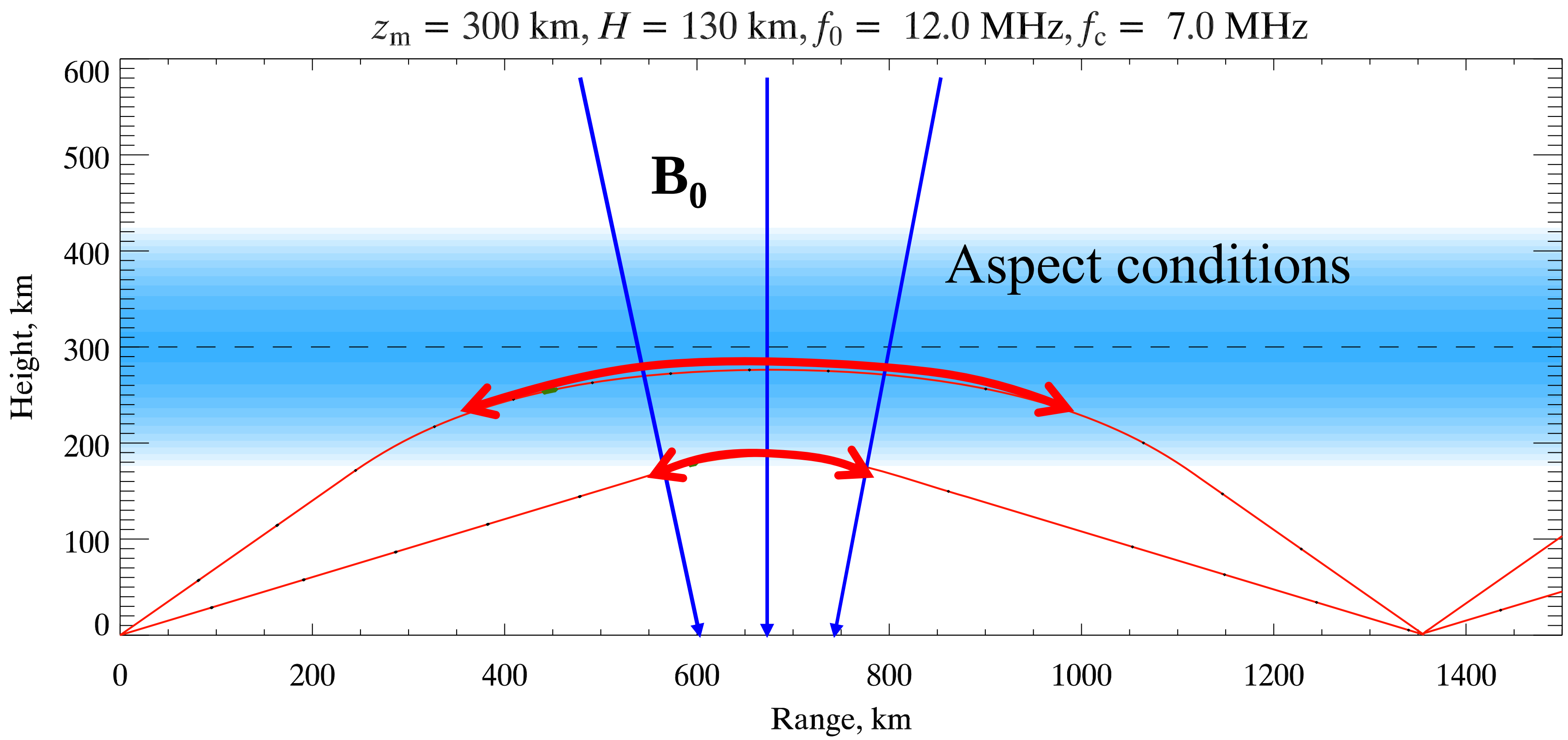
The best guess one can make…

Aspect conditions

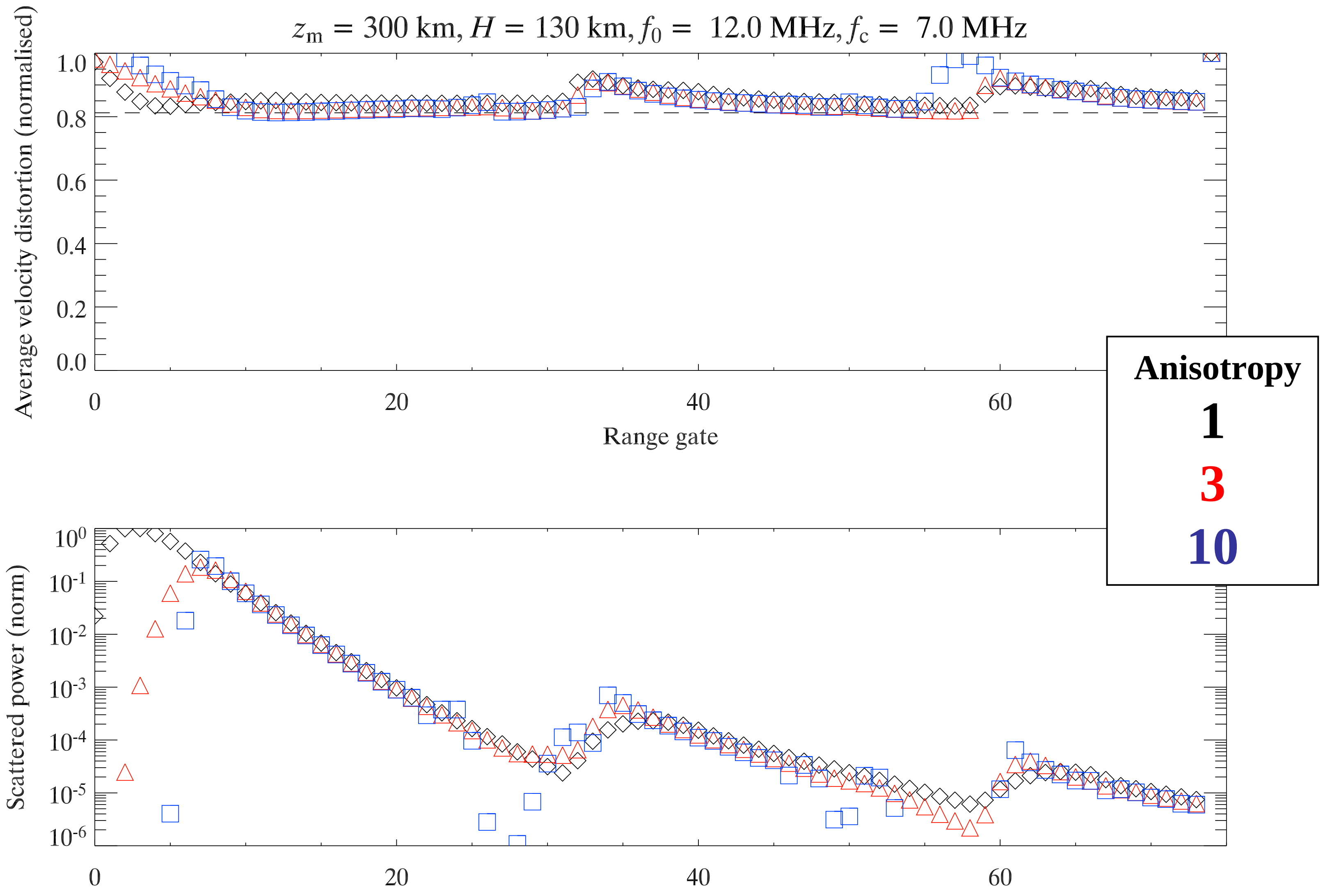




Upper and lower rays



Velocity distortion estimates



Conclusions

- By assuming scattering of HF waves at $\boldsymbol{V_p = c}$ FITACF underestimates the line-of-sight velocity magnitude by up to 25%, which causes the same distortion in the electric field magnitude.
- Ray-tracing analysis showed that the average velocity distortion over most of the range gates is close to its maximum possible magnitude at the electron density maximum
- As expected, refraction significantly reduces aspect sensitivity, and the velocity distortion is practically independent on the aspect ratio

Future directions

- Modification of the radar software to correctly account for the non-unity refractive index effect on velocity estimates based on IRI model estimates of f_{0F2}.
- Development of a more flexible ray-tracing software accounting for non-uniform ionosphere, magnetic field and spherical geometry.

IRI model (realistic ionosphere)